[40] R. LIPPERHEIDE, Nucl. Phys. **89,** 97 [1966].
[41] G. PASSATORE, Nucl. Phys. **A 110,** 91 [1968].
[42] R. LIPPERHEIDE and A. K. SCHMIDT, Nucl. Phys. **A 112,** 65 [1968].
[43] H. FIEDELDEY and C. A. ENGELBRECHT, Nucl. Phys. **A 128,** 673 [1969].
[44] I. AHMAD and M. Z. RAHMAN KHAN, Nucl. Phys. **A 132,** 213 [1969].
[45] M. L. GOLDBERGER, Phys. Rev. **74,** 1269 [1940].
[46] W. E. FRAHN and G. WIECHERS, Ann. Physics **41,** 442 [1967].
[47] M. L. GOLDBERGER and K. WATSON, Collision Theory, John Wiley & Sons, New York 1964.
[48] A. L. FETTER and K. M. WATSON, Adv. Theor. Physics **1,** 115 [1965].
[49] R. E. SCHENTER and B. W. DOWNS, Phys. Rev. **133 B,** 522 [1964].
[50] R. M. FRANK, J. L. GAMMEL, and K. M. WATSON, Phys. Rev. **101,** 891 [1956].
[51] A. E. GLASSGOLD, Rev. Mod. Phys. **30,** 419 [1958].

Mach's Principle — A Critical Review §

MICHAEL REINHARDT

Astronomische Institute der Universität Bonn

(Z. Naturforsch. **28 a,** 529—537 [1973]; received 19 October 1972)

After a short historical introduction it is discussed how far Mach's principle is incorporated into general relativity. The possible rôle of Mach's principle as a selection rule for the solutions of Einstein's field equations is summarized. Then follows a discussion of Mach's principle in theories of gravitation other than Einstein's, mainly the Brans-Dicke theory. Finally the experiments on the isotropy of inertial mass and their consequence for Mach's principle are described. The conclusion is that Mach's principle, though an extremely stimulating thought, has at present little claim to be a basic physical principle.

"They sought it with thimbles, they sought it with care;
They pursued it with forks and hope;
They threatened its life with a railway share;
They charmed it with smiles and soap."
LEWIS CARROLL: *"The Hunting of the Snark"*

I. Introduction

Mach's principle in its most general form can be stated as: *The inertial mass of a body is caused by its interactions with the other bodies in the universe.*

Despite, or perhaps because of, its vagueness and elusiveness it has intrigued many physicists and exerted a considerable influence on fundamental physics. It has played an important, if only heuristic, rôle in the construction of theories of gravitation, from Einstein's general relativity up to the Brans-Dicke theory. Nevertheless there is no consensus among physicists as to its standing in physics. The range of opinions extends from complete negation of the postulate, as being either physically irrelevant, or wrong, to the whole-hearted agreement in the sense that any theory of gravitation must comply with Mach's principle in order to be acceptable. The present paper tries to give a concise account of the concrete results which emerged in the long history of this controversy.

And its history is long indeed. It goes right back almost to the very beginnings of physics. NEWTON [1] himself asked the question of what singled out the inertial frames in which his first law holds. From his famous experiment of the rotating water pail [1, 2] he concluded that, since the shape of the water surface is independent of the motion of the bucket relative to the surface, it is not rotation with respect to other matter that determines the appearance of centrifugal and Coriolis forces but rotation relative to absolute space, which exists a priori and independently of the bodies in it. Thus it is acceleration relative to absolute space, or better its absence, that determines the preferred inertial frames. Leibniz, based on other philosophers back to Plato, rejected the idea of an absolute space [3]. He hold the relational view that "spatium est ordo coexistendi". For a discussion of the different notions on space hold by Newton and Leibniz see e. g. NORTH [4]. EULER [5] took the Newtonian point of view and influenced Kant's attitude [4], which in turn dominated the nineteenth century until Mach's critique of Newtonian mechanics. Mach had however an early precursor in

Reprint requests to Dr. M. REINHARDT, Observatorium Hoher List, *D-5568 Daun,* West-Germany

§ It is a pleasure to dedicate this paper to KONRAD BLEULER on his sixtieth birthday, whose widespread interest in fundamental questions of physics will always be remembered by all who have met him.

bishop BERKELEY [6]. Berkeley thought not only that the distant bodies of the universe (the 'fixed stars') provide a reference frame for the local inertial frames – e. g. in his critism of Newton's "Gedankenexperiment" of the two rotating bodies orbiting around each other in an otherwise empty universe, whose motion should be indetectable according to him. He also put forward the idea that the negative outcome of Newton's water bucket experiment was due to the negligible mass of the vessel, which makes only an imperceptibly small contribution to the inertial forces as compared to the distant masses of the fixed stars. MACH [7] elaborated Berkeley's ideas, in asking what happens to the water in the rotating bucket, if its walls are made more and more massive and thicker and thicker, until they are several miles htick. He expected an influence of the bucket on the inertial frame inside. Mach hold the view that the inertia of a body is produced by some interaction with all the other masses in the universe, so that the situation should be in every respect the same whether a body is rotating relative to the distant bodies of the universe, or the latter were rotating and the body was at rest. At least this seems to be how Einstein understood him. (For a critique ob Mach's positivistic ideas from the point of view of philosophy of science and semantics see BUNGE [3].) Einstein, who was impressed by Mach's ideas on inertia all his lifetime [8], actually tried to construct his theory of gravitation in such a way that it would incorporate them. As we will see later on he was only partially successful in this respect. However far from being laid aside as a tool that was useful in the construction of general relativity and whose usefulness expired with the accomplishment of the task, Mach's principle has since been discussed over and over, in the opinion of many physicists probably ad nauseam, to the present day. Perhaps the undiminished interest in Mach's principle is due to an intrinsic philosophical appeal. Perhaps to the authority of Mach and Einstein, who took it so seriously that he felt always unhappy about the incomplete incorporation of Mach's principle in general relativity [8].

II. An Observational Fact and Some Consequences of Mach's Principle

A number of authors [2, 9–12] stress the point that: The dynamically determined, e. g. by a gyroscope or a Foucault pendulum, local inertial frames coincide with the astronomically determined, i. e. in which the distant objects of the universe are nonrotating.

Or in the more cautious formulation of PIRANI [13]: "The local reference frames in which Newton's laws are approximately valid without the introduction of Coriolis or centrifugal forces are those which are approximately non rotating relative to the distant stars".

This is of course not a formulation of Mach's principle but a remarkable observational fact which may, or may not, have some deep significance. In general relativity this is simply a property of the most widely accepted cosmological solutions, the Friedmann universes. In these models the comoving system which provides the local inertial frame is also the one in which all distant bodies seem to move purely radially and isotropically. This does not hold for all general relativistic cosmological models, e. g. for universes with non-vanishing overall rotation. Thus in general relativity this is a coincidence [10, 13, 14], true only for a special class of cosmological models. Note that even then the coincidence of the frames can only be approximately true for an observer on the Earth, because of the de Sitter-Schouten effect. The geodesic precession in a circular geodesic orbit in Schwarzschild space implies that a gyroscope on the Earth will after a year not point to the same distant cosmic object as before.

Observations of distant galaxies imply that the relative rotation of the two frames must be smaller than $5 \cdot 10^{-9}$ rad/a [15, 16]. More stringent limits on the vorticity of the universe are provided by the large scale isotropy of the X-ray and microwave background, namely $\approx 10^{-13} - 10^{-15}$ rad/a and $\approx 10^{-16}$ rad/a respectively [17].

However for many authors it is not a sufficient explanation of this coincidence of frames that the initial conditions at the singularity of the universe, the big-bang, were such that it is now to a very good approximation Friedmannian. This leads to the formulation of Mach's principle given already in the introduction: Inertial reference frames are completely determined via some causal law by the distribution of all matter-energy in the observable universe [2, 8, 9, 10, 11, 18–26].

This assumption has four immediate consequences [8, 20, 22]:

(i) The inertial mass of a body should increase with the agglomeration of masses in its neighbourhood.

(ii) A body in an otherwise empty universe should have no inertia.

(iii) A body should experience an acceleration if nearby bodies are accelerated. The accelerating force should be in the same direction as the acceleration of the latter.

(iv) A rotating body should generate inside it a Coriolis force.

Effect (iii) occurs in general relativity [20, 22]. However, this does not necessarily imply that it is caused by a change of the inertial properties of the body. It can be interpreted as a change of the inertial frame due to the motion of gravitational masses [20], in the same sense as in the case of the presence of a gravitational mass at rest.

The same argument holds for effect (iv) [20], which also has a place in general relativity. THIRRING and LENSE [27, 28, 29] showed in the weak field approximation that inside an infinitely thin, slowly rotating shell a dragging of inertial frames takes place. There appear terms which are proportional to the angular velocity ω and ω^2. They were interpreted as Coriolis and centrifugal forces respectively. Some difficulties arose when BASS and PIRANI [30] could demonstrate that the allegedly centrifugal ω^2-terms did not vanish in the same frame as the ω-terms. They also included contributions from the elastic stress in the shell and found that if one chooses the density distribution in the shell in an appropriate way one can remove the ω^2-terms altogether. However COHEN and SARILL [31] managed to show that the ω^2-terms are solely quadrupole terms. The quadrupole moment arises as a consequence of the latitude dependent special relativistic mass increase with velocity in the shell. BRILL and COHEN [32, 33] investigated the same effect without the weak field approximation for different configurations of rotating masses in slow rotation. In accordance with some earlier results by HÖNL and SOERGEL-FABRICIUS [34, 35], LAUSBERG could show in a recent investigation [36] up to the first order in ω that in the static Einstein universe the dragging effect of a rotating shell on the inertial frame at the centre of the shell uniformly decreases with the distance, and increases with the thickness of the shell. Complete dragging is reached when the shell covers the whole universe. These results are often interpreted as being in favour of an incorporation of Mach's principle in general relativity. However, this is by no means a necessary interpretation [20].

EINSTEIN [8] thought that effect (i) occurred in general relativity. However this was based on a misinterpretation of a calculation performed in a special coordinate system as BRANS [37] was able to show. There are no observable effects in a laboratory from a spherically symmetric agglomeration of matter about it. This is a serious blow to Mach's principle.

Consequence (ii) has also no place in general relativity. A solution for empty space-time (the energy-momentum tensor $T_{\mu\nu} = 0$ everywhere) is the Minkowski space of special relativity where an infinitesimal test body has its usual inertia. Also in the case of the Schwarzschild space-time, i. e. of an isolated body in otherwise empty space, Birkhoff's theorem shows that by prescribing spherical symmetry we are bound to end up with the pseudo-euclidean Minkowski metric at infinity, so that again a test body has its full inertia however far from the only mass in the universe it may be.

To find a way out of this dilemma Einstein introduced the cosmological term Λ in his field equations. (The physical meaning of the Λ-term is rather obscure. According to EDDINGTON [38] and ZEL'DOVICH and NOVIKOV [20] it represents a locally unobservable energy density of the vacuum.) Einstein thought that the field equations with $\Lambda > 0$ would have no solutions for $T_{\mu\nu} = 0$, i. e. that there would be no inertia in the absence of matter, in accordance with Mach's principle. However, after DE SITTER [39] found a solution for the modified field equations for $T_{\mu\nu} = 0$, which represents an expanding universe with ever increasing velocity, EINSTEIN [40] dropped the Λ-terms as superfluous and no longer justified. One should note that objections have been raised to consequence (ii) as making no sense, since it has no operational basis [13] (we cannot empty the whole universe), and represents an extrapolation of the theory over too many orders of magnitude [10]. Nevertheless this question of solutions for empty space has worried many authors, as we will presently see.

III. Mach's Principle as a Selection Rule in General Relativity

Thus Mach's principle is not fully incorporated in general relativity. To make this more clear we will look at the following formulation of Mach's principle:

The metric tensor $g_{\mu\nu}$ (the gravitational and inertial field) must be determined completely by the energy momentum tensor $T_{\mu\nu}$ (the distribution of matter-energy) [8, 13, 19, 24, 41].

Now it is clear from Einstein's field equations that the $T_{\mu\nu}$ influence the $g_{\mu\nu}$. However, since Einstein's field equations are second order differential equations, the $g_{\mu\nu}$ cannot be determined alone by the $T_{\mu\nu}$, and one needs boundary conditions to obtain a unique solution for given $T_{\mu\nu}$. This situation is familiar from electrostatics. One has to supplement Poisson's equation by suitable boundary conditions in order to obtain a unique solution. Usually one demands that the potential vanishes at infinity.

The necessity of boundary conditions, or of symmetry conditions or initial values in the cosmological problem, has been regarded by some authors [9, 42] as a substitute for Newton's "absolute space" in general relativity. On the other hand this problem induced HÖNL [43] and later on WHEELER [24, 44] and GILMAN [45] to regard Mach's principle as a selection rule to separate admissible solutions of Einstein's field equations from inadmissible ones. I. e. they hold that it does not make sense to require that Mach's principle is a consequence of the field equations of gravitation, but that it is an independent constraint on the solutions.

In their early work [46, 47] HÖNL and DEHNEN used the criterion that the relativity of rotation should hold, i. e. that the same Coriolis and centrifugal forces should arise whether a test body rotates and the universe is at rest or vice versa. They found that this requirement is fulfilled in the static Einstein universe and in the Friedmann and Lemaître universes, but not in the Minkowski and de Sitter space-times. They excluded open models for reasons of conceptual difficulties connected with the "actually infinite". For a thorough discussion of this difficulty and how it can be overcome by means of Cantor's transfinite cardinals see e. g. NORTH [4].

Later HÖNL and DEHNEN [48, 49] used the criterion that the inertial frames should be completely determined by the sum of the energy-momentum densities of matter and gravitational field, plus symmetry conditions to classify cosmological solutions in Machian and anti-Machian. They claimed that in this sense the static Einstein universe, the Friedmann models, the de Sitter world, the GÖDEL universe [50], as well as the OZSVÁTH-SCHÜCKING solution [51] were Machian, whereas the Schwarzschild solution, the Minkowski world, and the OZSVÁTH-SCHÜCKING vacuum solution [52], filled with gravitational radiation, would be anti-Machian. As in their previous investigations they held that all closed space-times are automatically Machian and excluded open models for the same rather philosophical reason stated above.

Their work was criticized by EHLERS and SCHÜCKING [53] mainly on the following grounds. Because of the (weak) principle of equivalence, gravitation must appear in the equation of motion on the side of inertia, not on the side of force as in the Hönl-Dehnen formulation. Second, Hönl and Dehnen argue in a vicious cycle: the force is calculated from sources which already encompass the $g_{\mu\nu}$, hence the force field. Moreover Ehlers and Schücking could show that all Friedmann-models admit regular source-free Maxwell fields, so that with the Hönl-Dehnen definition of Mach's principle all Robertson-Walker models would be anti-Machian, even the Einstein universe. For a reply to this critique which, however, could not dispel the doubts raised see HÖNL and DEHNEN [12].

WHEELER [24] has made a very serious attempt to formulate Mach's principle as a selection rule imposed on the boundary conditions for Einstein's field equations. As a suitable selection principle he proposes that the geometry of space-time must be closed and free of singularities. To enforce compatibility with this requirement for the Schwarzschild case, Wheeler proposed the lattice universe where many masses are spaced uniformly through a closed universe. Another proposed remedy is to place a mass in the TAUB universe [54], a closed universe filled with gravitational waves only. Of course embedding in a radiation-filled universe of the Tolman type or in a closed Friedmann universe would do as well. In both cases mentioned before the geometry close to the mass is nearly described by the Schwarzschild solution. The trouble with the lattice universe is that Mach's principle breaks down in the limit of infinite cell size. Also there is an ambiguity in the interpretation of the geometry by an observer, since the convergence in the limit of an infinitely great lattice universe is non-uniform.

Wheeler has stressed the necessity to provide a mathematically well defined formulation of Mach's principle in order to avoid circular reasoning of the following type. In the case of the Taub universe one has to specify not only the distribution of matter but also of gravitational radiation in order to determine the geometrry however, in general relativity gravitational radiation is nothing else but an aspect of geometry. Wheeler proposed the following formulation of Mach's principle:

"The specification of a sufficiently regular closed three- dimensional geometry at two immediately succeeding instants, and of the density and flow of mass-energy, is to determine the geometry of space-time, past, present, and future, and thereby the inertial properties of every infinitesimal test particle".

He has given an expression of it in mathematical form by his "condensed intrasurface variational principle" (see Eq. (62) [24]). Note that here the boundary conditions have been expressed as an initial value problem, which is more appropriate for the investigation of cosmological models. Wheeler finds that the variational integral has in general not a finite and well defined value for open spaces. Therefore he concludes that the validity of Mach's principle requires a closed universe. This is important since it makes his formulation of Mach's principle testable at least on a cosmological scale. Wheeler has not discussed the effect of additional masses in the vicinity of a test particle on its inertia, which would provide a local test. Also the standing of Gödel's universe with its closed timelike world lines with regard to Wheeler's formulation of Mach's principle is not quite clear. For a critique of Wheeler's investigation see HELLER [55, 56].

Another way of attack to test which solutions of the field equations satisfy Mach's principle in the formulation given in the beginning of this section is to get rid of the boundary conditions by replacing the field equations by integral equations. In Newtonian theory we have the field formulation where the Laplacian of the gravitational potential is given by the density distribution, and the particle formulation where the potential is given by the sum of the contributions of all the masses. The first requires boundary conditions and is strictly parallel to Einstein's field equations. An attempt to do the second in general relativity is extremely complicated by the fact that the field equations are non-linear, so that "summing up" the contributions to $g_{\mu\nu}$ from sources at different points in space-time becomes rather awkward.

A first attempt in this direction was made by SCIAMA [57] in his vector theory of inertia and gravitation. In this theory the inertial field is treated as arising from radiative interaction resulting from the acceleration of distant matter, whereas gravitation appears as the static part of this interaction. Sciama's theory is linear and has some other shortcomings as well, e. g. the loss of general covariance. For a critique see DAVIDSON [58] and DICKE [22]. SCIAMA himself realized that this theory could only be a first step, and that a satisfactory theory would have to be of a tensor type, and undertook first steps in this direction [25, 59].

HOYLE and NARLIKAR [60–63] encouraged by the Wheeler-Feynman particle formulation of electrodynamics tried to construct a direct interparticle action theory of gravitation. Although they did not fully succeed – see the critique by DESER and PIRANI [64] – their work helped to clarify the situation and stimulated further research. Using their idea of summing up the sources $T_{\mu\nu}$ in an explicitly linear way in actual space-time, which is however implicitly non-linear since it depends on the sources also via the propagator for the geometry, and the theory of retarded bi-tensors developped by SYNGE [65] and DE WITT and BREHME [66], AL'TSHULER [67] and LYNDEN-BELL [26] formulated Einstein's equations in generally covariant integral form. The non-linearity of the integral representation is ensured by using as kernel a retarded bi-tensorial Green's function depending in its turn on the metric tensor. Their results have been further elaborated by SCIAMA et al. [68]. This representation is the tensor analogy of Kirchhoff's integral in electrodynamics. The inertial-gravitational potential (the metric tensor $g_{\mu\nu}$ at some point in space-time is given by a volume integral over the sources $T_{\mu\nu}$ in the past light cone plus a surface integral. The surface integral is interpreted as the contribution from sources outside the volume of integration plus the source-free contribution. GILMAN [45] used this formulation of Einstein's equations for a classification of cosmological models according to Mach's principle, which he states in the form: "Physically admissible are only those space-times that are entirely source generated."

In order to do so he separated the source-free contribution in the surface integral and checked for which solutions this part vanishes. Gilman restricted the analysis to globally hyperbolic models, i. e. those without closed timelike world lines, since the integral over the past light cone is not well defined, if this restriction is not made. This excludes e. g. the Gödel universe from the beginning. Also all models with $\Lambda \neq 0$ are regarded as non-Machian, for Λ must be treated in this representation as a source term[45, 68], yet the cosmological constant is in the opinion of the authors not a Machian source term. It is quite ironical that Λ which was introduced by Einstein in the hope to make his theory Machian is here regarded as intrinsically non-Machian. Non-Machian also, according to this analysis, are the Minkowski and Schwarzschild space-times, the homogeneous anisotropic KANTOWSKI-SACHS-THORNE models[69, 70], all Schwarzschild type solutions, i. e. which are flat at infinity, e. g. the KERR-solution[71], and all vacuum cosmological solutions, i. e. those with $T_{\mu\nu} = 0$ everywhere. On the other hand all non-empty Robertson-Walker models with physically admissible pressure are probably Machian. However, since all Friedmann models admit regular source-free Maxwell fields[53], even these universes should be non-Machian in the sense of Gilman. An improvement of Gilman's treatment, which does not include gravitational effects of matter outside the past light-cone and source-free contributions, has been recently achieved by RAINE[71a]. Retaining the conditions of global hyperbolicity and $\Lambda = 0$, Raine finds that Gilman's method is correct for Robertson-Walker models, since contributions from outside the particle horizon cancel each other for reasons of symmetry. Raine postulates two conditions for a space-time to be Machian:

(i) The metric tensor of a Machian space-time can be represented uniquely as a generalized inverse functional of the Riemann tensor, modulo a term which can be reduced to zero by a gauge transformation; (ii) The curvature is to be determined by a linear superposition of the effects of material sources. According to Raine the first condition is necessary, both together are sufficient. Raine finds that (i) all empty space-times are non-Machian; (including asymptotically flat and plane-wave space-times); (ii) all conformally flat, non-flat, space-times satisfy the second Mach condition (a trivial consequence is that all Robertson-Walker space-times are Machian); (iii) the homogeneous anisotropic Bianchi models are non-Machian; (iv) the Kantowski-Sachs models are probably non-Machian; (v) the perfect fluid homogeneous rotating cosmologies are non-Machian; (vi) the inhomogeneous pressure free, perfect fluid Bondi models with Robertson-Walker type singularities are Machian, those with Heckmann-Schücking or Kasner type singularities are not. In my opinion Raine's work is the most important step made so far in this direction, and it might indeed be the answer to Mach's principle as a selection rule. Of course observational cosmology will have the last word as to whether Mach's principle in Raine's formulation is valid or not.

Before we go on to Mach's principle in other theories of gravitation, let us mention briefly some other discussions of this topic in general relativity. KÁROLYHÁZY[72] also investigated Mach's principle as a selection rule for cosmological models. The standing of PACHNER's paper[73] in relation to general relativity is not quite clear in view of his statement that "the vanishing of the matter tensor does not yet signify the absolute absence of matter". ROSEN's discussion[23] of the increase with time of the inertial mass in the comoving system in closed Robertson-Walker models implies a variation of the gravitational constant with time ($\dot{G} \neq 0$). This is a clear violation of the strong principle of equivalence on which general relativity rests. It seems that he gets his result by keeping the expansion rate of the universe (the Hubble parameter) constant with time (conservation of the 3-momentum). He states himself that another formulation of this result is that the inertial mass is constant and the momentum decreases (which is the usual interpretation). The situation seems to be similar in GÜRSEY's discussion[74] of Mach's principle in general relativity, who also arrives at $\dot{G} \neq 0$ in closed universes by keeping the space curvature fixed. TANGHERLINI[75] and HORÁK[76–81] revived the argument of the Λ-term with regard to Mach's principle. Tangherlini assumed that Λ depends on the mean matter density in the universe. Horák considers a static universe with $\Lambda \neq 0$ and holds that the velocity of light is determined by Λ, the gravitational constant, and the mean matter density in the universe. To make his model compatible with the observed expansion of the universe he assumes a large number of expanding and contracting regions in a static universe. However the radius of the static universe as calculated from his formula (4.11)[80] for a mean matter density of $2 \cdot 10^{-29}\,\mathrm{g/cm^3}$ is only $2 \cdot 10^9\,\mathrm{pc}$. This is certainly not enough to incorporate a large number of contracting and expanding regions without a violation of the observed redshift of galaxies, even if we disregard the quasars. Also the observed isotropy of the microwave background is unaccounted for in his model.

IV. Mach's Principle in Theories of Gravitation Different from General Relativity

Modifications of general relativity are often motivated by Mach's principle in one or another form. Thus Hoyle's steady-state theory [9, 82–84] was constructed among other reasons around the assumption that the coincidence of local inertial frames with the non-rotating frame of distant objects has to be explained by some process, and not by initial conditions as in general relativity. In the BONDI and GOLD version [85] of the steady state theory based on the "perfect cosmological principle" this coincidence is a by-product. In Hoyle's modification of Einstein's field equations by the addition of the scalar C-field, the continuous creation of matter smoothes out irregularities in the universe as it expands, while rotation, if it is present, vanishes with the increasing age of the universe [9]. We will not discuss here the later remodifications of the steady-state theory, nor its present observational status. Let us merely say that for a number of observational reasons, the discovery of the microwave background being the most important, the steady-state theory is not widely accepted at present.

Another departure from general relativity built around Mach's principle is the Brans-Dicke theory, which is a scalar-tensor theory of gravitation. Mach's principle is introduced by interpreting the scalar field as an advanced wave integral over all matter [8-]. For a detailed discussion of the theory see DICKE [22, 87]. One of the central relations in this theory is: $GM/Rc^2 \approx 1$, where G is the gravitational constant, c the velocity of light, M and R the mass and radius of the observable universe respectively. This relation was formulated earlier by SCIAMA [2, 57] and HÖNL [43] as the Machian feedback condition. It allows two interpretations (if it is not merely a numerical coincidence!): (i) G is a constant and the mass distribution M/R is fixed by the field equations and/or the boundary conditions; (ii) G is variable and a function of M/R. The first interpretation leads in the framework of the Brans-Dicke theory to a variation of the inertial mass (adjustment by the scalar field), with the scalar field as a "matter field" so that Einstein's field equations are formally valid. By a transformation of units the theory becomes identical with the JORDAN theory [88]. It is interesting to note that Jordan in building his theory rejected Mach's principle altogether [89]. In the second form of the theory the gravitational constant is variable with time, but the inertial mass of a test body is constant. DICKE [86] claims that the Brans-Dicke theory is Machian in the sense that (i) there are no solutions at all for an empty space; (ii) space closes about a localized mass configuration. The closure of space about an isolated mass is achieved by the scalar field [22]. Point (i) has been proved to be wrong, since there exist vacuum solutions for gravitational waves which are geodesically complete [87]. NARIAI [90, 91] made some investigations of the Green's function required for an integral formulation of the Brans-Dicke theory. KATZ [11] stresses that the situation concerning boundary conditions in the Brans-Dicke theory is essentially the same as in general relativity, i. e. they are necessary to enforce Mach's principle. TOTON [21, 92] has pointed out another defect of the Brans-Dicke theory with regard to Mach's principle. In general relativity the self-energy of a neutral point particle is zero, whereas the self-energy of the point-electron is equal to the magnitude of its charge [93]. Toton could show that the same holds in the Brans-Dicke theory, and especially that the mass of the electron (its self-energy) remains constant if the value of the scalar field, which is interpreted as the Machian interaction with distant matter, is changed.

Another critical discussion of the relation of the Brans-Dicke theory with regard to Mach's principle has been given by HELLER [55, 56]. It seems that Mach's principle is no more incorporated in the Brans-Dicke theory than in general relativity. However, the Brans-Dicke theory predicts sizeable differences in the classical tests of general relativity, the light deflection (or time delay) and in the perihelion precession. It gives also a different value for the dragging of inertial frames in the field of a rotating body [94]. Thus there may well be a difference in the standing of both theories concerning Mach's principle. At least the predictions of the Brans-Dicke theory against those of general relativity are locally testable, which is in the eyes of the author the main advantage of the Brans-Dicke theory as compared to locally virtually untestable theories like the steady-state theory. One should perhaps mention the recent attempt by FIRMANI [95] to modify general relativity, or the Brans-Dicke theory, in such a way that the inertial mass of a body increases, if masses are agglomerated in its neighbourhood. It is a two tensor theory of gravitation. Its relation to general relativity and to the experiment discussed in the next section are not quite clear.

V. A Test of Mach's Principle?

It is clear that if Mach's principle has any claim to be scientific it must be testable, as has been stressed particularly by BONDI [19] and BUNGE [96]. COCCONI and SALPETER [97, 98] have proposed such an experiment. If the inertial mass of a body is determined by the interaction with distant matter, our eccentric position in the galaxy should produce an anisotropy i. e. a directional dependence of the inertial mass. Cocconi and Salpeter assume that in first approximation the contribution Δm to the inertial mass m by the mass M at distance r is proportional to r^{ν}, with $0<\nu<1$, for, if $\nu<0$ then the more distant a given amount of mass, the greater would be its contribution to Δm, which is not reasonable. On the other hand, if $\nu>1$, then nearby matter, e. g. the sun, would dominate the determination of the inertial mass, which is incompatible with celestial mechanics. Thus COCCONI and SALPETER [97, 98] predicted an anisotropy of the inertial mass due to our eccentric position in the galaxy (its mass assumed to be concentrated at the centre) of $\Delta m/m = 3\cdot 10^{-10}$ for $\nu=0$ and $2\cdot 10^{-5}$ for $\nu=1$.

Several experiments have been performed in order to test for a mass anisotropy during the earth rotation in a day. The constancy of a quartz crystal oscillator with respect to the hyperfine structure transition frequency of a caesium clock [99] gave an upper limit to the mass anisotropy of about 10^{-9}. Observations of the relative frequencies of the two Zeeman transitions [100] of atomic oxygen gave $\Delta m/m<10^{-10}$. A study of the nuclear energy levels of Fe^{57} by means of the Mössbauer effect [101] provided an upper limit of $5\cdot 10^{-16}$. The most sensitive test for mass anisotropy so far was obtained from an ordinary nuclear magnetic resonance of Li^7. This experiment gave $\Delta m/m<10^{-22}$ (BELTRAN-LOPEZ et al. [102]) and $<5\cdot 10^{-23}$ (DREVER [103]). For a summary of these experiments see HUGHES [104]. Thus the isotropy of the inertial mass is very well established. Actually the Hughes-Drever experiment is the most precise null-experiment ever performed.

Is this a deadly blow for Mach's principle?

According to ZEL'DOVICH and NOVIKOV [20] and WEBER [105] this is so. However, DICKE [86] and EPSTEIN [106] found a way to save Mach's principle by postulating that there should be no locally observable effect because inertial anisotropy should be present in the same way for all particles and fields, although the Hughes-Drever experiment still excludes two-tensor theories of gravitation [86] and a variability of the fine-structure constant [107]. However one should note that if Dicke's interpretation of Mach's principle is correct, it is a Pyrrhic victory to some extent, since Mach's principle becomes then locally untestable.

VI. Conclusion

Mach's principle is not incorporated into general relativity. Its usefulness as a selection rule for solutions of Einstein's field equations is not really established (Section III). It seems that other theories of gravitation, like the Brans-Dicke theory (Section IV), are not much better off with regard to Mach's principle. And the only test so far proposed, that of the anisotropy of inertial mass, came out negative, which means either that Mach's principle is wrong or that it is locally untestable. Should it be buried then? Should we return to absolute space as SYNGE [65] and FOCK [108] hold? Of course it is not absolute space in the sense of Newton, since space-time is influenced by the masses in it. But one can call it absolute in the sense of Minkowski, since the curvature invariants that characterize space are measurable in an absolute way, i. e. coordinate independent.

If one decides to bury Mach's principle, as the author is inclined to do, it should be buried with honours, since it has stimulated so much research and thereby produced a gamut of important results.

This study was started while the author was a European Fellow of the Royal Society at the Institute of Theoretical Astronomy in Cambridge.

1 I. NEWTON, Mathematical Principles of Natural Philosophy,, ed. F. CAJORI, University of California Press, Berkeley 1947.
2 D. W. SCIAMA, The Physical Foundations of General Relativity, Doubleday, London 1969.
3 M. BUNGE, Amer. J. Phys. **34**, 585 [1966].
4 J. D. NORTH, The Measure of the Universe, Clarendon, Oxford 1965.
5 L. EULER, Opera omnia, Soc. Sci. Nat. Helv., Zürich 1960.
6 G. BERKELEY, A Treatise Concerning the Principles of Human Knowledge, sec. 110—117 (1710) ; De Motu, sec. 52—65 (1721) ; The Works of George Berkeley, Clarendon, Oxford 1901.

[7] E. MACH, Die Mechanik in ihrer Entwicklung historisch kritisch dargestellt, Brockhaus, Wiesbaden 1904.
[8] A. EINSTEIN, The Meaning of Relativity, Princeton University Press, Princeton 1955.
[9] F. HOYLE and J. V. NARLIKAR, Proc. Roy. Soc. London A **273**, 1 [1963].
[10] H. BONDI, Cosmology, University Press, Cambridge 1952.
[11] A. KATZ, Nuovo Cim. **51 B**, 502 [1967].
[12] H. HÖNL and H. DEHNEN, Z. Phys. **206**, 492 [1967].
[13] F. A. E. PIRANI, Helv. Phys. Acta Suppl. **4**, 198 [1956].
[14] W. H. MCCREA, Nature London **230**, 95 [1971].
[15] L. I. SCHIFF, Rev. Mod. Phys. **36**, 510 [1964].
[16] G. M. CLEMENCE, Rev. Mod. Phys. **29**, 2 [1957].
[17] A. M. WOLFE, Astrophys. J. **159**, L 61 [1970].
[18] A. EINSTEIN, S. B. preuss. Akad. Wiss. **1917**, p. 142.
[19] H. BONDI, Assumption and Myth in Physical Theory, University Press, Cambridge 1967.
[20] YA. B. ZEL'DOVICH and I. D. NOVIKOV, Relativistskaya Astrofizika, Nauka, Moscow 1967.
[21] E. T. TOTON, Phys. Rev. Lett. **21**, 1401 [1968].
[22] R. H. DICKE, Chapt. 7 in "Gravitation and Relativity", ed. CHIU and HOFFMANN, Benjamin, New York 1964.
[23] N. ROSEN, Ann. Physics **35**, 426 [1965].
[24] J. A. WHEELER, Chapt. 15 in "Gravitation and Relativity, ed. CHIU and HOFFMANN, Benjamin, New York 1964.
[25] D. W. SCIAMA, Rev. Mod. Phys. **36**, 463 [1964].
[26] D. LYNDEN-BELL, Mon. Not. Roy. Astr. Soc. **135**, 413 [1967].
[27] H. THIRRING, Phys. Z. **19**, 33 [1918].
[28] H. THIRRING and J. LENSE, Phys. Z. **19**, 156 [1918].
[29] H. THIRRING, Phys. Z. **22**, 29 [1921].
[30] L. BASS and F. A. E. PIRANI, Phil. Mag. **46**, 850 [1956].
[31] J. M. COHEN and W. J. SARILL, Nature London **228**, 849 [1970].
[32] D. R. BRILL and J. M. COHEN, Phys. Rev. **143**, 1011 [1966].
[33] J. M. COHEN and D. R. BRILL, Nuovo Cim. B **568**, 209 [1968].
[34] C. SOERGEL-FABRICIUS, Z. Phys. **159**, 541 [1960].
[35] H. HÖNL and C. SOERGEL-FABRICIUS, Z. Phys. **1663**, 571 [1961].
[36] A. LAUSBERG, Astron. & Astrophys. **3**, 150 [1969].
[37] C. BRANS, Phys. Rev. **125**, 2194 [1962].
[38] A. S. EDDINGTON, Sci. Progr. **34**, 225 [1939].
[39] W. DE SITTER, Mon. Not. Roy. Astr. Soc. **78**, 3 [1917].
[40] A. EINSTEIN, S. B. preuss. Akad. Wiss. **1931**, p, 235.
[41] W. PAULI, Relativitätstheorie, Boringhieri, Torino 1958.
[42] G. BECK, Chapt. 4 in "Handbuch d. Physik IV", Springer-Verlag, Berlin 1929.
[43] H. HÖNL, Z. Naturforsch. **8 a**, 2 [1953].
[44] J. OORT and J. A. WHEELER, 11th Solvay Conf., Brussels, p. 49, 1959.
[45] R. C. GILMAN, Phys. Rev. D **2**, 1400 [1970].
[46] H. HÖNL and H. DEHNEN, Z. Phys. **166**, 544 [1962].
[47] H. DEHNEN, Z. Phys. **166**, 559 [1962].
[48] H. HÖNL and H. DEHNEN, Ann. Physik **11**, 201 [1963].
[49] H, HÖNL and H. DEHNEN, Ann. Physik **14**, 271 [1964].
[50] K. GÖDEL, Rev. Mod. Phys. **21**, 447 [1949].
[51] J. OZSVÁTH and E. SCHÜCKING, Nature London **193**, 1168 [1962].
[52] J. OZSVÁTH and E. SCHÜCKING, in "Recent Developments in General Relativity", Pergamon Press, New York 1962.
[54] J. EHLERS and E. SCHÜCKING, Z. Phys. **206**, 483 [1967].
[54] A. H. TAUB, Ann. Math. **53**, 472 [1951].
[55] M. HELLER, Acta Phys. Polon. B **1**, 123 [1970].
[56] M. HELLER, Acta Phys. Polon. B **1**, 131 [1970].
[57] D. W. SCIAMA, Mon. Not. Roy. Astr. Soc. **113**, 34 [1953].
[58] W. DAVIDSON, Mon. Not. Roy. Astr. Soc. **117**, 212 [1957].
[59] D. W. SCIAMA, Proc. Roy. Soc. A **273**, 484 [1963].
[60] F. HOYLE and J. V. NARLIKAR, Proc. Roy. Soc. A **277**, 1 [1964].
[61] F. HOYLE and J. V. NARLIKAR, Proc. Roy. Soc. A **282**, 178 [1964].
[62] F. HOYLE and J. V. NARLIKAR, Proc. Roy. Soc. A **282**, 184 [1964].
[63] F. HOYLE and J. V. NARLIKAR, Proc. Roy. Soc. A **282**, 190 [1964].
[64] S. DESER and F. A. E. PIRANI, Proc. Roy. Soc. A **288**, 133 [1965].
[65] J. L. SYNGE, Relativity, the General Theory, North Holland, Amsterdam 1960.
[66] B. DEWITT and R. BREHME, Ann. Phys. **9**, 220 [1960].
[67] B. L. AL'TSHULER, Zh. Exp. Teor. Fiz. **51**, 1143 [1966].
[68] D. W. SCIAMA, P. C. WAYLEN, and R. C. GILMAN, Phys. Rev. **187**, 1762 [1969].
[69] R. KANTOWSKI and R. K. SACHS, J. Math. Phys. **7**, 443 [1966].
[70] K. S. THORNE, Astrophys. J. **148**, 51 [1967].
[71] R. P. KERR, Phys. Rev. Lett. **11**, 237 [1963].
[71a] D. J. RAINE, to be published.
[72] F. KÁROLYHÁZY, Acta Phys. Hungar. **17**, 31 [1964].
[73] J. PACHNER, Phys. Rev. **132**, 1837 [1963].
[74] F. GÜRSEY, Ann. Physics **24**, 211 [1963].
[75] F. R. TANGHERLINI, Nuovo Cim. Suppl. **20**, (Ser. X) 1 [1961].
[76] Z. HORÁK, Bull. astr. Inst. Czech. **14**, 119 [1963].
[77] Z. HORÁK, Phys. Lett. **28 A**, 332 [1968].
[78] Z. HORÁK, Czech. J. Phys. B **19**, 703 [1969].
[79] Z. HORÁK, Phys. Lett. **30 A**, 83 [1969].
[80] Z. HORÁK, Bull. Astr. Inst. Czech. **21**, 96 [1970].
[81] Z. HORÁK, Czech. J. Phys. B **20**, 1194 [1970].
[82] F. HOYLE, Mon. Not. Roy. Astr. Soc. **108**, 372 [1948].
[83] F. HOYLE, Mon. Not. Roy. Astr. Soc. **109**, 396 [1949].
[94] F. HOYLE, Mon. Not. Roy. Astr. Soc. **120**, 266 [1960].
[85] H. BONDI and T. GOLD, Mon. Not. Roy. Astr. Soc. **108**, 252 [1948].
[86] R. H. DICKE, "Experimental Relativity" in "Relativity Groups and Topology", ed. DEWITT and DEWITT, Gordon and Breach, New York 1964.
[87] W. KUNDT and M. TRÜMPER, Mainzer Veröff. IV, 1962.
[88] P. JORDAN, Z. Phys. **157**, 112 [1959].
[89] P. JORDAN, Schwerkraft und Weltall, Vieweg, Braunschweig 1952, p. 138.
[90] H. NARIAI, Progr. Theor. Phys. **40**, 49 [1968].
[91] H. NARIAI, Progr. Theor. Phys. **40**, 657 [1968].
[92] E. T. TOTON, J. Math. Phys. **11**, 1713 [1970].
[93] R. ARNOWITT, S. DESER, and C. W. MISNER, Chapt. 7 in "Gravitation: An Introduction to Current Research", ed. L. WITTEN, Wiley, New York 1962.
[94] D. R. BRILL, Z. Naturforsch. **22 a**, 1336 [1967].
[95] C. FIRMANI, Astrophys. Space Sci. **13**, 128 [1971].
[96] M. BUNGE, Amer. J. Phys. **36**, 166 [1968].
[97] G. COCCONI and E. E. SALPETER, Nuovo Cim. **10**, 646 [1958].
[98] G. COCCONI and E. E. SALPETER, Phys. Rev. Lett. **4**, 176 [1960].
[99] L. ESSEN, J. V. L. PARRY, and J. MCA. STEELE, Proc. IEE **107 B**, 229 [1960].
[100] H. E. RADFORD and V. W. HUGHES, Phys. Rev. Lett. **4**, 399 [1960].
[101] C. W. SHERWIN, H. FRAUENFELDER, E. L. GARWIN, E. LÜSCHER, E. MARGULIES, and R. N. PEACOCK, Phys. Rev. Lett. **4**, 399 [1960].
[102] V. BELTRAN-LOPEZ, H. G. ROBINSON, and V. W. HUGHES, Bull. Am. Phys. Soc. **6**, 424 [1961].
[103] R. W. P. DREVER, Phil. Mag. **6**, 683 [1961].
[104] V. W. HUGHES, Chapt. 6 in "Gravitation and Relativity", ed. CHIU and HOFFMANN, Benjamin, New York 1964.
[105] J. WEBER, General Relativity and Gravitational Waves, Interscience, New York 1961.
[106] S. T. EPSTEIN, Nuovo Cim. **16**, 587 [1960].
[107] P. J. PEEBLES and R. H. DICKE, Phys. Rev. **127**, 629 [1962].
[108] V. A. FOCK, Teoria prostranstva, vremeni i tyagotenia, Fismatgis, Moscow 1955.